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The interdependence of inventory management and retail shelf management

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Abstract *Acknowledges that the effect of displayed inventory on retail sales is widely recognized in the logistics, marketing and operations management literature and has been empirically verified. However, neither the marketing literature (shelf-space allocation models) nor the operations management literature (inventory control models) has appropriately modeled this effect. The displayed-inventory news-vendor problem is developed and analyzed, utilizing a simple model to illustrate the interdependencies between the inventory and space-allocation decisions. The model is then extended to the multi-item case, which can be incorporated as part of a comprehensive shelf-management system.*

Introduction

It is a well-established phenomenon that displayed inventory has a positive effect on the sales of many retail items. Nearly half a century ago, Whitin (1957) indicated:

> . . . for retail stores the inventory control problem for style goods is further complicated by the fact that inventory and sales are not independent of one another. An increase in inventories may bring about increased sales of some items.

Wolfe (1968) presented empirical evidence of this relationship for style goods and observed that:

> . . . within the selling season unit sales of each style are proportional to the amount of inventory displayed.

Schary and Becker (1972) noted that distribution has been traditionally viewed as an enabling factor, but stated that another role of distribution is to stimulate demand, that product availability can be a stimulus to a purchase decision. Larson and DeMarais (1999) and Dubelaar *et al.* (2001) called this "psychic stock" and stated that there are a number of retail logistics implications, including pushing inventory forward in the distribution channel to maintain retail display stock levels. Wang and Gerchak (2001) recently investigated a decentralized manufacturer-retailer supply chain that recognizes the positive dependence of demand on the quantity displayed.

This paper examines this relationship between the displayed inventory level and the demand of an item to gain practical insight into the interdependence of inventory management and retail shelf management. First, a review of the relevant literature is presented. Then, a simple, but practical, model is

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developed, which is used to characterize the specific relationship between sales and displayed inventory. Finally, the basic model is extended to the multiple-item case and to the situation incorporating the effect of other variables.

Background

Marketing researchers and professionals have taken advantage of the relationship between sales and displayed inventory through the development and application of shelf-space allocation models (Anderson and Amato, 1974; Borin *et al.*, 1994; Bultez and Naert, 1988; Corstjens and Doyle, 1981; Zurfryden, 1986). These models formulate the demand rate as a function of the space allocated to a particular item – and sometimes the allocation of competing, substitute, or complementary items – frequently using the multiplicative, constant-elasticity, functional form:

$$d_s = \alpha s^{\beta} \quad \alpha > 0,\ 0 < \beta < 1, \tag{1}$$

where d_s is the demand rate of a particular product, s is the shelf space allocated to the product, and α and β are the scale and shape parameters of the demand function, respectively. While these models have been valuable in incorporating the effect of displayed inventory on the demand of an item, this functional form of demand implicitly assumes the shelves are always fully stocked, or at least assumes the customer knows, and is influenced by, the space allocated to a particular product even if the product is not present on the shelf. Using this rationale, the demand for a product with 12 depleted facings (no product on the shelf) would be greater than if it only had six depleted facings. A more appropriate formulation would be to model the demand as a function of the amount of inventory actually displayed to the customer.

A class of inventory control models has been introduced in the operations management literature that incorporates the effect of inventory on sales (Baker and Urban, 1988a; Bar-Lev *et al.*, 1994; Chung *et al.*, 2000; Mandal and Phaujdar, 1989; Padmanabhan and Vrat, 1995). In these inventory-level-dependent, or stock-dependent demand models, the demand rate is explicitly formulated as a function of the inventory level; thus, as the inventory level decreases, the demand rate decreases. While several different functional forms have been presented in the literature, the polynomial formulation is frequently used, since it is characterized by diminishing returns (the marginal increase in the demand rate decreases as the inventory level increases) as well as the richness of the function and its intrinsic linearity (see Baker and Urban (1988a) for a discussion of the advantages of this functional form):

$$d_i = \alpha i_t^{1-\beta} \quad \alpha > 0,\ 0 < \beta < 1, \tag{2}$$

where i_t is the instantaneous inventory level at time t. The underlying assumption of these models, however, is that all of the product is displayed; that is, there is no back-room inventory that is used to store product before it is shelved and available to the customer. While this may be appropriate for some organizations, many retailers would have a back-room inventory or warehouse that receives the product, which is not visible to the customer, and would not have an impact on the demand rate.

Recently, attention has begun to focus on the situation in which a portion of the total order quantity is displayed to the customer. In the context of product assortment and shelf-space allocation, Urban (1998) developed a continuous-review model in which the demand is a function of the displayed inventory. The order is received into backroom storage, and the display is continuously replenished by the backroom inventory. Ray *et al.* (1998) have also developed a continuous-review, displayed-inventory model, but assumed the displayed inventory is periodically replenished from a second warehouse, with transportation costs involved in getting the inventory from the warehouse to the display area.

One particular type of inventory model is known as the news-vendor, or single-period, problem (e.g. Bramel and Simchi-Levi, 1997, p. 180). There has been growing interest in the news-vendor problem due, in part, to its applicability in retailing (Khouja, 1999). This model is characterized by a single item under consideration, with only one opportunity to acquire the items each period, that being at the beginning of the selling period. The items procured in one period cannot be used to satisfy demand in subsequent periods – relevant for products that have a short shelf life (e.g. many grocery items) or a short demand life (e.g. periodicals). Additionally, Smith and Agrawal (2000) indicated that an inventory model with a fixed cycle for replenishment and no lead time can be modeled as a news-vendor-type problem. Retail stores commonly replenish inventories on a fixed cycle with one week being a typical cycle length, although this may vary by type of product. Also, retailers with electronic data interchange (EDI) systems are sufficiently responsive such that their inventory system may be modeled in this manner. Thus, Smith and Agrawal claim that news-vendor models are appropriate for major retail chains that use an EDI system to transmit sales data.

Baker and Urban (1988b) and Urban and Baker (1997) investigated deterministic, news-vendor models in which the demand rate is a function of the instantaneous inventory level. While the traditional news-vendor model is trivial in the deterministic case – as the exact quantity demanded during the period would be ordered – this is not so for the situation with an inventory-level-dependent demand rate, since it may be more effective to end the period with a positive inventory level. It may increase profitability by ordering an amount greater than the demand during the period and realizing the increased sales due to the larger inventory level, while incurring the holding cost on some unsold product remaining at the end of the period. Gerchak and Wang (1994) developed a stochastic model that assumes that the demand rate is dependent on the starting inventory; thus, the demand remains constant and does not decrease as the inventory level decreases (*à la* shelf-space allocation models).

The displayed-inventory news-vendor model

In this section, the inventory-level-dependent demand news-vendor model is generalized to reflect explicitly the effect that the observed inventory has on the demand of many retail products. At the beginning of the period, the inventory level will likely be greater than the allocated shelf space; thus, the display area (the amount of shelf space allocated to a particular product) will be filled, with the remainder placed into a warehouse or other back-room storage area. As long as the inventory level exceeds the shelf-space allocation, the quantity displayed and, consequently, the demand rate will be constant. At the point at which the inventory level is not sufficient to stock fully the allocated shelf space (i.e. when the back-room inventory is depleted), the displayed inventory level will decrease, resulting in a decreased demand rate.

Assumptions and model formulation

The assumptions of the proposed model are the same as that of the classical news-vendor model, except it is assumed that the demand rate is a deterministic function of the displayed inventory. A deterministic model will be investigated due to the ability to determine a closed-form solution, which will be used to develop insight into the problem. While a stochastic model may better reflect a typical retail situation, this simple model will allow the study of the interdependencies between the marketing (shelf-space allocation) and inventory (order quantity) decisions. It is also assumed that the display area is continually kept fully stocked as long as there is sufficient inventory, and there is an associated cost of display space. There are two decision variables under the retailer's control: the order quantity and the shelf-space allocation.

The notation used for the model is as follows:

- q = order quantity;
- s = shelf-space allocation (number of facings);
- i_t = inventory level at time t (back-room inventory plus displayed inventory);
- ϕ = displayed inventory level;
- d_ϕ = demand rate, a function of the displayed inventory level;
- p = retail selling price (revenue) per unit;
- c = acquisition cost per unit ($p > c$);
- h = holding cost per unit remaining at the end of the period ($c > -h$);
- v = shelf-space cost per unit (facing), based on the allocated shelf space;
- T = length of the time period.

The holding cost, h, can be interpreted more generally by including any cost of removing the item from inventory at the end of the period less any revenue received if there is salvage value of the item; thus, this value could be negative

(Johnson and Montgomery, 1974, p. 45). In the logistics literature (see Ballou, 1999), the space costs are also considered to be part of the holding costs; they are considered separately here to distinguish between the inventory and the shelf-space allocation decisions.

To maintain consistency with the existing literature, the polynomial functional form of demand is used:

$$d_\phi = \alpha\phi^{1-\beta} \quad \alpha > 0,\ 0 < \beta < 1, \tag{3}$$

where $\phi = \min\{s, i_t\}$. As previously mentioned, the period will likely begin with fully-stocked shelves; thus, the demand rate is constant. All of the demanded items are being replenished from the back-room inventory. During this time, the inventory level will decrease at a constant rate (see Figure 1). Once the back-room inventory has been depleted (at time τ), subsequent demand will result in the displayed inventory level decreasing which, in turn, will result in a lower demand rate. Thus, the inventory level will decrease at a decreasing rate. The mathematical representation of the inventory level over time can be expressed as:

$$i_t = \begin{cases} q - \alpha s^{1-\beta_t} & \text{for } 0 \le t \le \tau = \dfrac{q-s}{\alpha s^{1-\beta}} \\ \left[s^\beta - \alpha\beta(t-\tau)\right]^{1/\beta} & \text{for } \tau \le t \le \omega = \dfrac{q + \frac{(1-\beta)s}{\beta}}{\alpha s^{1-\beta}}. \\ 0 & \text{for } t \ge \omega \end{cases} \tag{4}$$

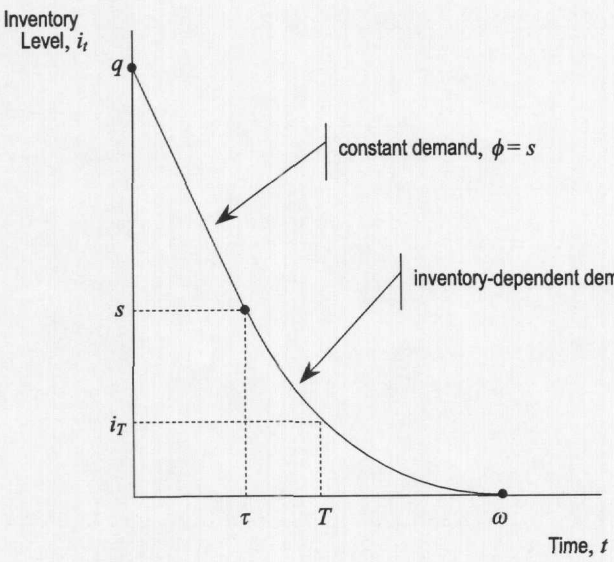

Figure 1. Graphical representation of how the inventory level changes over time

Solution methodology

The objective will be to maximize the total profit over the planning horizon. As with inventory-level-dependent demand inventory models, cost minimization is not appropriate, since the decision variables directly affect the demand rate. If the objective were to minimize costs, it would be preferable to keep the demand as low as possible, by maintaining lower inventories, to avoid the acquisition and holding costs. Realistically, though, achieving higher sales for profitable items is obviously beneficial to the retailer, so it may be preferable to end the period with a positive inventory level to realize the higher demand. The profit for each period can be expressed as the gross revenue less the acquisition, shelf-space and holding costs:

$$\pi = p(q - i_T) - cq - vs - hi_T. \tag{5}$$

The following properties of this problem concerning the inventory level at the end of the period can be exploited in evaluating this function. First, an order will never be placed such that the ending inventory is greater than the shelf-space allocation. The reason for maintaining higher inventory levels is to increase the sales during the period; however, beyond this point, the demand rate is constant, so there is no advantage further increasing the order quantity. Second, an order will never be placed such that the inventory level is depleted before the end of the period (assuming it is a profitable product). Thus, the inventory function need only be evaluated over the interval $\tau \leq T \leq \omega$, and the profit function can be expressed as:

$$\pi = (p - c)q - vs - (p + h)\left[s^\beta - \alpha\beta\left(T - \frac{q - s}{\alpha s^{1-\beta}}\right)\right]^{1/\beta}. \tag{6}$$

Using traditional calculus techniques (details are included in the appendix), a closed-form solution can be found:

$$s = \left[\frac{\alpha\beta\varphi T}{\varphi(1 - \theta^\beta) + \beta v}\right]^{1/\beta} \tag{7}$$

$$q = \alpha s^{1-\beta}T - s\left[\frac{(1 - \beta) - \theta^\beta}{\beta}\right] \tag{8}$$

where

$$\theta = \left(\frac{p - c}{p + h}\right)^{1/1-\beta}$$

and

$$\varphi = (1 - \beta)(p - c).$$

While it has not been proven that equation (6) is a quasiconcave function (i.e. a unimodal function such that any local maximum is known to be a global maximum) for all values of the parameters, all examples investigated have demonstrated this characteristic; furthermore, it would be straightforward to test this for a given set of parameters. It can also be shown that as β approaches one (i.e. the demand is not a function of the displayed inventory), the optimal order quantity approaches αT (the demand realized during the period) and the allocated shelf space approaches zero (since displayed inventory has no effect on sales).

Numeric example

To illustrate the behavior of this type of model, consider the following example in which the demand follows the functional form, as described in equation (3) with the values of the parameters as follows:

Selling price, $p = \$18$ per unit.

Acquisition cost, $c = \$12$ per unit.

Shelf-space cost, $v = \$4$ per unit.

Demand-scale parameter, $\alpha = 0.76$.

Demand-shape parameter, $\beta = 0.40$.

Time period, $T = 7$ days.

Holding cost, $h = -\$2$ per unit remaining at the end of the time period.

From equations (7) and (8), the optimal order quantity, $q^* = 16.97$ units, and the optimal shelf-space allocation, $s^* = 8.04$ units, can be determined, resulting in a total profit of $\pi^* = \$44.59$. Over 9 percent of the initial inventory will remain in stock at the end of the period, $i_T = 1.57$ units. The shelves will be fully stocked for roughly half of the period, $\tau = 3.37$ days; therefore, the inventory models with inventory-level-dependent demand would not be representative of this situation during the first half of the period since only eight units are displayed, and the shelf-space allocation models would not be representative during the last half of the period since fewer than eight units will be displayed.

The difference in operating objectives between marketing (revenue maximization) and operations management (cost minimization) "may lead to a fragmentation of interest in, and responsibility for, logistics activities" (Ballou, 1999). To achieve greater sales levels, a retailer may wish to maintain fully-stocked shelves throughout the period, which creates maximum product visibility and exposes the customers to as much stock as possible. This "full shelf merchandising" policy (Larson and DeMarais, 1999) will result in a solution of $q = 10.70$ units, $s = i_T = 2.19$ units, and $\pi = \$20.43$, generating a profit less than half of the optimal solution ($\pi/\pi^* = 45.81$ percent). Noting that this is a deterministic model, an operations manager may be inclined to order such that the inventory is depleted precisely at the end of the period to avoid leftover product and incur no holding costs. In this situation, the only decision variable is the shelf-space allocation since, once s is determined, the value of the order quantity can be calculated such that $i_T = 0$. For the parameters given above, the solution is $s = 2.63$ units with $q = 5.56$ units and $\pi = \$22.83$, a profit just over half of the optimal solution ($\pi/\pi^* = 51.20$ percent). Obviously, using rules of thumb to determine the appropriate decision variables may result in poor decisions.

Managerial implications

In this section, the interdependence between the operations management (order quantity) and the marketing (shelf-space allocation) decisions will be investigated. The sensitivity of the solution to changes in the model parameters will also be analyzed, again emphasizing the inventory/shelf management relationship.

Relationship between order quantity and allocated shelf space

Obviously, it would be expected that some interdependency between the order quantity and the shelf-space allocation would be found. The marketing literature (see e.g. Drèze *et al.*, 1994, p. 304) reports that a frequently used method to determine retail space for each item is to allocate space in approximate proportion to its sales, direct product profitability (unit contribution to overhead and profits), or operating costs; thus, the shelf-space allocation would increase approximately linearly with demand. From an inventory-control perspective, a positive relationship is also expected between the demand rate and the order quantity, although (due to the familiarity of the economic order quantity) the order quantity is generally expected to increase roughly in proportion to the square root of demand. Therefore, as the demand rate of a product increases, it would be expected that the allocated shelf-space would increase at a greater rate than the order quantity.

In fact, the relationship between the optimal values of the shelf-space allocation, s, and the order quantity, q, is linear and can be expressed as follows:

$$\frac{s}{q} = \frac{\varphi}{\varphi + v} = \frac{(1-\beta)(p-c)}{(1-\beta)(p-c)+v}. \tag{9}$$

In other words, the optimal value of the shelf-space allocation is a given fraction of the order quantity. This ratio is a factor of three items:

(1) As the effect of displayed inventory on demand increases ($\beta \to 0$), both s and q increase and the ratio approaches $(p-c)/[(p-c)+v]$. On the other hand, as the effect of displayed inventory decreases, this ratio approaches zero, since the allocated shelf space approaches zero, and the order quantity approaches αT.

(2) As the cost of the display area goes to zero ($v \to 0$), the order quantity approaches the allocated shelf space; that is, there is no need for back-room inventory as all of the product can be economically displayed on the shelf. Conversely, as the cost of the display increases, the s/q ratio approaches zero, as increasingly less shelf space would be utilized for this item.

(3) The order quantity and the shelf-space allocation are similarly affected by changes in the profitability of the product. As the item becomes less profitable ($p \to c$), both q and s (as well as the ratio) approach zero; as the item becomes more profitable, both q and s increase and the s/q ratio approaches one.

It is interesting to note that the holding cost, h, does not appear in equation (9). In fact, both the order quantity and space allocation are equally affected by changes in the holding cost; that is, as h increases, both q and s decrease proportionally. Another interesting relationship between the two variables is the following ratio of the shelf-space allocation and the inventory remaining at the end of the period:

$$\frac{i_T}{s} = \theta = \left(\frac{p-c}{p+h}\right)^{1/1-\beta}, \qquad (10)$$

where $(p-c)/(p+h)$ is the solution to the traditional, stochastic news-vendor model. It is obvious from this relationship that the shelf-space allocation, s, and the ending inventory, i_T, are equally affected by changes in the shelf-space cost, v; that is, as v increases, both i_T and s decrease proportionally.

Frequently, the allocated shelf space of a particular item may be based on other issues – such as the competitive environment, pressure from the manufacturer, etc. – rather than an analytic, economic modeling approach. Figure 2 illustrates the sensitivity of the model in changes to the values of the decision variables

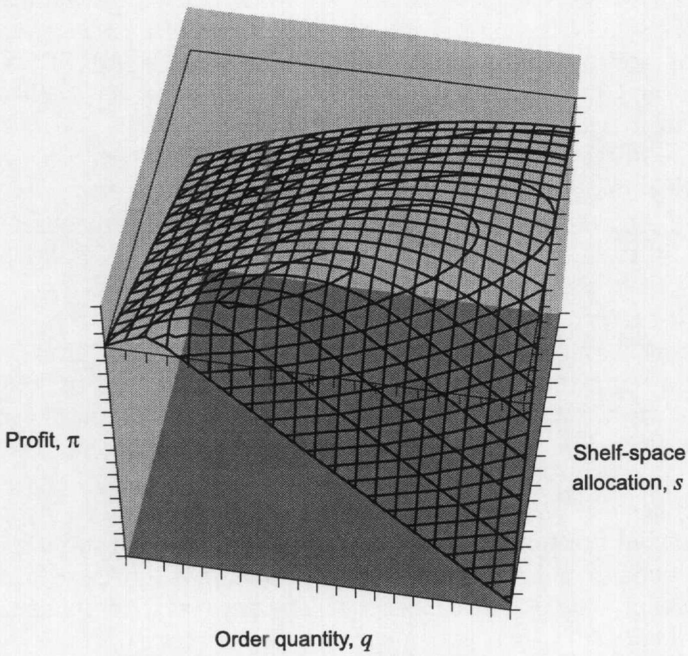

Figure 2. Sensitivity of profit to changes in the decision variables

(using the data from the example in the previous section). The relationship between the order quantity and the allocated shelf space is clearly seen in the figure. As a solution deviates from the optimal solution, there is little loss of profit as long as it moves along the "ridge" on the graph. Essentially, a line could be drawn on the graph with a slope, from equation (9), of $1 + v/\phi$ along which small losses would be realized. On the other hand, if a solution were to move away from that ridge, the profit quickly decreases. For example, if both q and s are increased 25 percent above their optimal values, then the profit decreases by less than 5 percent; however, if q is increased by 25 percent and s is decreased by 25 percent, then a profit reduction of nearly 70 percent is realized. Thus, the model is insensitive to changes in the order quantity and space allocation as long as the relationship in equation (9) is maintained; otherwise, the model is very sensitive to changes in the decision variables. This further reiterates the need to coordinate marketing and operations management decisions.

Estimating cost parameters

While much has been written about estimating the traditional inventory costs, relatively little emphasis has been placed on estimating the display cost. Of course, the inventory models with inventory-level-dependent demand do not incorporate such a cost, and while some of the shelf-space allocation papers do, they are not explicit as to what should be included; that is, what incremental costs are incurred for each additional facing. One approach that may be taken is to base the incremental shelf-space cost on the "pay-to-stay" fee or other slotting allowance (e.g. Bloom *et al.*, 2000) that is based on allocating a certain amount of space. Drèze *et al.* (1994) note that store occupancy costs are about \$20 per square foot for dry grocery shelf space (considerably more if refrigeration or freezer space is needed); this type of cost could be used as a proxy for shelf-space costs. Thus, while the selling price and the unit cost may be fairly easy to identify, it is likely to be much more difficult to find a precise value of the shelf-space cost, v.

Figure 3 illustrates the sensitivity of the model with respect to errors made in estimating the shelf-space cost, again using the example in the previous section. As long as the previously discussed relationship between the order quantity and the allocated shelf space is maintained, there is little deviation from the optimal profit; a +300 percent error results in a reduction in profit of roughly 50 percent. Only as the applied value of v approaches zero (–100 percent error) does the profitability begin to deteriorate substantially. Figure 3 also illustrates this sensitivity of the shelf-space cost for different values of β. It is apparent the model is more sensitive to errors in estimating the shelf-space cost as the effect of displayed inventory increases (i.e. as β decreases).

Estimating demand parameters

The polynomial functional form of demand (equation (3)) can be transformed into a linear equation, $y = a + bx$, where $y = \ln(d_\phi)$, $a = \ln(\alpha)$, $b = 1 - \beta$, and $x = \ln(\phi)$. This allows the use of regression analysis for estimating the parameters, using existing operational data. However, sufficient data may not be available

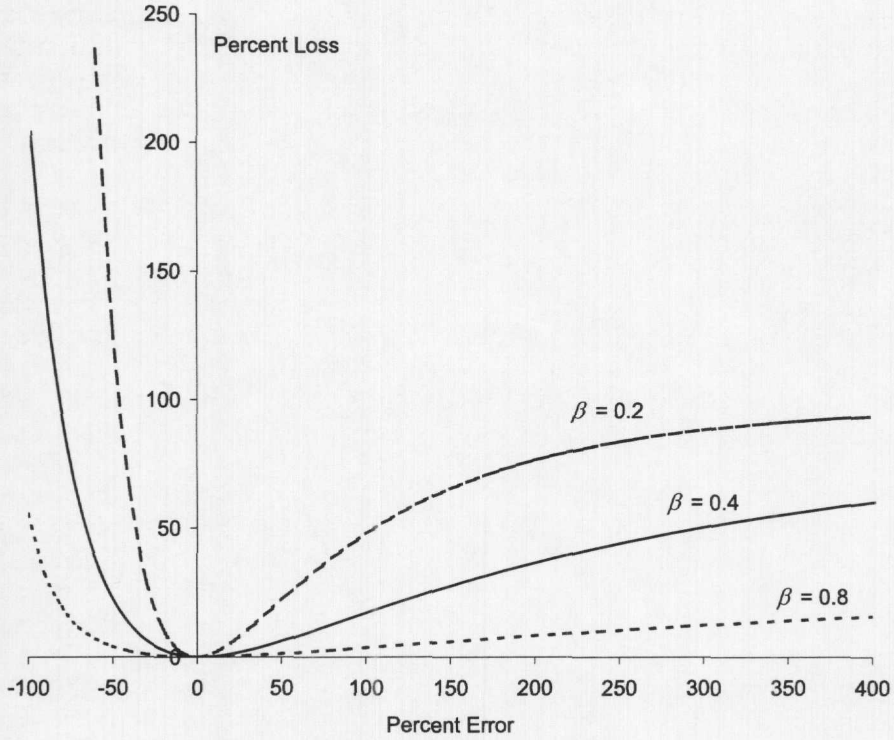

Figure 3. Deviation from optimal profit due to errors made in estimating shelf-space cost

on historical levels of shelved vs. back-room stock or shelf-space allocations. If this is the case, in-store experiments may be required to get reliable parameter estimates; however, the cost, labor and disruptions required for such experimentation can be substantial.

To determine the consequences of inaccurate estimates of the demand parameters, Figure 4 presents the sensitivity analysis on the values of the scale (α) and shape (β) parameters. It is obvious from this figure that errors made in estimating these parameters can result in substantial losses. In particular, as the estimated value of β becomes smaller, indicating an increase in the expected effect of displayed inventory on demand, then the loss incurred quickly increases. Similarly, large estimated values of α will result in substantial penalties. On the other hand, small estimates of α and large estimates of β will result in reduced levels of allocated shelf space and order quantities; thus, the impact on profits is less profound. Of course, during the estimation of the parameters, if the value of β were underestimated, it would likely be compensated for with an offsetting error of α.

Extension to the multi-item case

To be part of a comprehensive shelf-management system, the displayed-inventory news-vendor model must be simultaneously applied to all of the items within a category. However, retail shelf space is not an unlimited

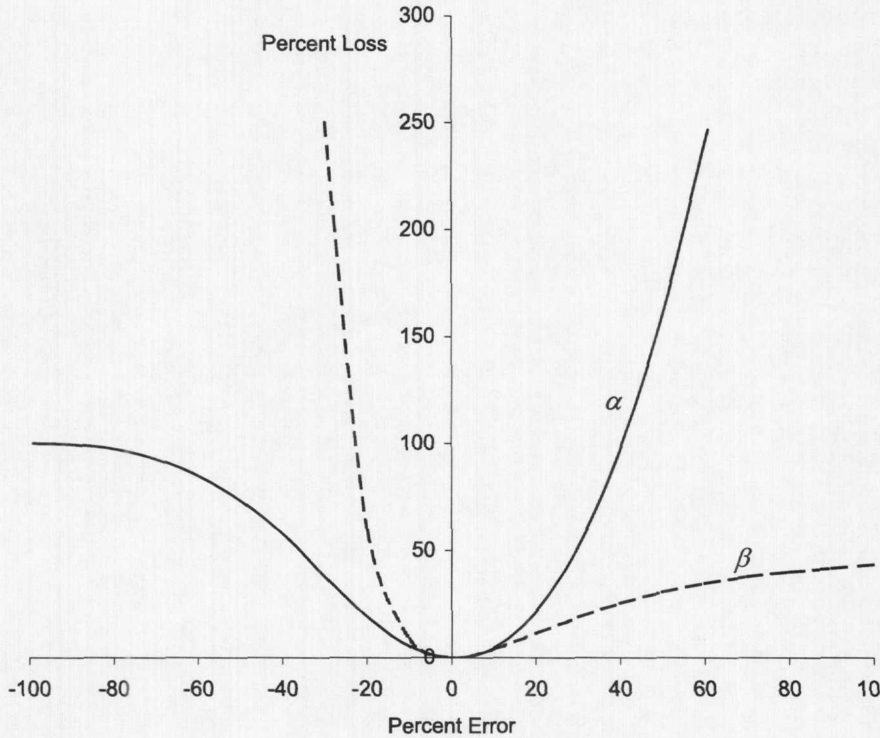

Figure 4.
Deviation from optimal profit due to errors made in estimating demand parameters

resource, and independent allocation decisions for each item will likely call for more space than is available. Thus, the problem is to identify an appropriate allocation for each item that does not exceed the available shelf space in the aggregate. As will be shown, the manipulation of the shelf-space cost makes the extension of the displayed-inventory news-vendor problem to the constrained, multiple-item situation (a.k.a. the "news-stand" problem, Lau and Lau, 1996) fairly straightforward.

A basic shelf-management model

Consider the situation in which there are a set of products – each of which follows the displayed-inventory news-vendor framework – competing for limited shelf space. The demand rate of each of the items is assumed to follow the polynomial functional form of equation (3). The demand parameters and selling price, unit cost, and holding cost may be different for each product, while the shelf-space cost, v, and the length of the period, T, are the same. There is a limited amount of shelf space available for the entire set of products.

In an empirical analysis of the effect of allocated shelf space on sales, Desmet and Renaudin (1998) disregarded cross-elasticities (the sales responsiveness of an item on the space allocated to another item) for several reasons. First, they noted that the direct space elasticities are typically higher than the cross-elasticities between items and will reflect the greatest impact of shelf space on demand. They also indicated that cross-elasticities would tend not to be as significant in stores such as supermarkets, which contain a large number of products, most of which do not easily substitute for or complement one another. Finally, they acknowledged the difficulty in obtaining reliable estimates of cross-elasticities in stores with a large number of product categories. Zurfryden (1986) also questioned the practical use of cross-elasticities, stating, "the consideration of 'cross-elasticities' at the individual product level would be impossible in practice due to the overwhelming number of cross-elasticity terms that would need to be estimated". Indeed, for a store with n items, a total of $n(n-1)$ cross-elasticity parameters would need to be estimated, requiring a great deal of data or experimentation. Thus, it is assumed that the product demands are independent and cross-elasticities between items in the category do not need to be incorporated. While this may restrict the generality of the analysis to some extent, it renders the application of the model more practical and more amenable to solution, particularly in this setting, since the inventory level of each of the items will be depleted to its allocated shelf space, which affects its demand rate, at different times during the period.

To solve the displayed-inventory news-stand problem, the focus is on the one aspect that characterizes this problem, the display area. It is proposed that for a given shelf-space cost, if the resulting "optimal" space requirements of the individual items, $\sum s_i$, is greater than the available shelf space, then the shelf-space cost has been underestimated. By increasing the fee charged for the shelf space to the level at which the resulting sum of the individual allocations equals the available shelf space, an appropriate solution to the constrained problem can be found. Thus, a simple univariate search technique can be used to find the appropriate value of v and the resulting solution to the overall problem. A valuable byproduct of this approach is that the resulting shelf-space cost, v', can be interpreted as reflecting the shadow price of the problem (actually, Lagrange multiplier would be the more accurate term since it is a non-linear problem); in other words, v' represents the incremental value per unit of available shelf space.

Numeric example

To illustrate the process, consider the six-product example shown in Table I, using the demand parameters and space constraint data from Borin *et al.* (1994) as well as the necessary cost and time data. If each of the items were analyzed individually using equations (7) and (8), the total required shelf space, $\sum s_i$, would be about 41 facings, considerably more than the 24 facings available.

Any univariate search technique can be used to identify the solution to the constrained news-stand problem, since the total required shelf space increases monotonically with respect to the shelf-space cost. This approach alleviates the need for a non-linear programming algorithm. For this example, a shelf-space cost of 3.43 provides a solution utilizing 24 facings (the solution is also provided in Table I). Again, this value $(v' - v = 3.43 - 3 = 0.43)$ represents the increase in total profit for each unit increase of the available display area.

Item, i	Demand parameters Scale α_i	Shape β_i	Input data: Selling price p_i	Acquisition cost c_i	Holding cost h_i	Initial solution (using $v = 3$): Order quantity q_i	Allocated shelf space s_i	Final solution (using $v = 3.45$): Order quantity q_i	Allocated shelf space s_i
1	28.53	0.1532	1.62	0.90	1.00	88.8	15.0	49.9	7.5
2	23.62	0.2273	2.70	1.50	0.50	58.3	13.8	41.5	8.8
3	25.59	0.2089	1.98	1.10	1.50	34.6	6.5	23.8	4.0
4	22.40	0.2143	1.80	1.00	0.00	20.4	3.5	13.5	2.1
5	15.62	0.2955	2.25	1.25	–1.00	8.2	1.6	6.0	1.0
6	10.50	0.3104	3.15	1.75	–0.50	2.9	0.7	2.3	0.5
				$\sum s_i =$		41.1		24.0	

Length of time period, $T = 1/3$
Shelf-space cost, $v = 3$
Available shelf space = 24 facings
(each item requires one facing per unit)

Note: The values for the order quantity, q_i, and allocated shelf space, s_i, of each item are determined from equations (7) and (8)

Table I.
Input data and solution for the multi-item example

Incorporating other marketing variables

While the displayed-inventory news-vendor problem is useful in investigating the relationship between the inventory and shelf-space allocation decisions, it is often desirable to extend the analysis to include price as a decision variable as well as promotional expenditures or other controllable variables that could be used to influence the target market (Lilien *et al.* (1992) present a set of such marketing-mix variables, focusing on the interaction among the variables). Maintaining the constant elasticity model, the demand function would be expressed as:

$$d_{\phi,p,x} = \alpha\phi^{1-\beta}p^{-\varepsilon}\prod_i x_i^{\gamma_i} \quad \alpha > 0, 0 < \beta < 1, \varepsilon > 1, 0 < \gamma < 1, \tag{11}$$

where ε is the price elasticity of demand (precluding inelastic demand schedules) and x_i represents any other variables that may be relevant (with shape parameters of γ_i). Any additional variables would likely incur associated costs and would need to be included in the profit function; for example, if advertising expenditures, r, were incorporated into the model, the profit for a particular item would be:

$$\pi = (p - c)q - vs - r - (p + h)\left[(1-\beta)s^{\beta} + \frac{\beta q}{s^{1-\beta}} - \alpha\beta p^{-\varepsilon}r^{\gamma}T\right]^{1/\beta}. \tag{12}$$

The decision variables would now be the order quantity, space allocation, price and advertising expenditures for each item, and the objective would be to maximize the sum of the individual profit functions.

To generalize the problem even further, various constraints may be included in the model, in addition to the available shelf space. For example, retailers frequently stipulate a minimum amount of displayed stock for a particular item to make the display look attractive, to provide exposure and awareness for a new product, or to maintain minimum exposure for a prestige product ($s \geq S^-$). Also, a limit may be placed on the allocated shelf space of a product at a later stage of its life cycle to keep the display up to date ($s \leq S^+$). Restrictions on the amount of on-hand inventory may be necessary due to storage, budget or availability limits ($\sum q \leq Q^+$). Alternatively, there may be minimum purchase requirements from a supplier ($q \geq Q^-$). Constraints on price mark-up (perhaps due to competitive factors, $p \geq \lambda^- c$, $p \leq \lambda^+ c$) or on advertising expenditures (within a given budget, $\sum r \leq R$, or as a constant percentage of sales, $r \leq f[q - i_T]$) can also be included.

With this formulation, a closed-form solution can no longer be identified; the problem is now a non-linear programming problem with mn variables, where m is the number of decision variables for each item ($m = 4$ in equation (12)) and n is the number of items in the category. Separable programming – a non-linear programming method in which the non-linear functions are approximated by piecewise linear functions – could be used for this application, since the profit and constraint functions, as described above, can be separated with appropriate transformations (see Baker and Urban (1988a), for a comparable application of separable programming to an inventory system with inventory-level-dependent demand). For large problems, various heuristics could be used – simulated annealing (Borin *et al.*, 1994) and genetic algorithms (Urban, 1998) have been successfully applied to shelf-space allocation problems.

Conclusion

One of the major decisions made by retail managers is the allocation of scarce shelf space among competing products. The availability of electronic checkout scanners allows the rapid collection and analysis of data necessary to improve allocation decisions. Thus, recent research has focused on category management models, and the development and use of commercial shelf-management software has become increasingly widespread. A second recurring problem faced by retailers is the management of inventory. Strategic retailing initiatives, such as quick response and efficient consumer response, in conjunction with the increasing use of technology, including electronic data interchange and bar coding, have brought inventory planning to the attention of retailers. Consequently, researchers and software developers have also placed emphasis on the management and control of retail inventories. While retail shelf management and inventory management are obviously related problems, no research has been conducted that focuses on identifying the interrelationship between these two retail concerns. As noted by Ellram *et al.* (1999):

> . . . for these technologies and the supply chain management concept to be truly effective . . . retailers must resolve their internal conflicts, integrating internal functions.

The purpose of this paper is to gain insight into the interdependencies of the inventory management and the retail shelf management policies. A simple, but practical, model that realistically represents the effect of displayed inventory on demand is developed to provide a framework to analyze the relationship between these variables. A closed form solution is found, from which it is discovered that there is a linear relationship between the optimal order quantity and allocated shelf space. Thus, as long as this direct relationship is preserved, there is little penalty of deviating from optimality; if it is not, the solution quickly deteriorates. Finally, the basic model is generalized by considering the multi-item situation, which can be used as part of a comprehensive shelf-management program or can be used as a stand-alone system for small retailers, and extending the analysis to incorporate other variables.

References

Anderson, E.E. and Amato, H.N. (1974), "A mathematical model for simultaneously determining the optimal brand-collection and display-area allocation", *Operations Research*, Vol. 22 No. 1, pp. 13-21.

Baker, R.C. and Urban, T.L. (1988a), "A deterministic inventory system with an inventory-level-dependent demand rate", *Journal of the Operational Research Society*, Vol. 39 No. 9, pp. 823-31.

Baker, R.C. and Urban, T.L. (1988b), "Single-period inventory dependent demand models", *Omega*, Vol. 16 No. 6, pp. 605-7.

Ballou, R.H. (1999), *Business Logistics Management: Planning, Organizing, and Controlling the Supply Chain*, 4th ed., Prentice-Hall, Upper Saddle River, NJ.

Bar-Lev, S.K., Parlar, M. and Perry, D. (1994), "On the EOQ model with inventory-level-dependent demand rate and random yield", *Operations Research Letters*, Vol. 16 No. 3, pp. 167-76.

Bloom, P.N., Gundlach, G.T. and Cannon, J.P. (2000), "Slotting allowances and fees: schools of thought and the views of practicing managers", *Journal of Marketing*, Vol. 64 No. 2, pp. 92-108.

Borin, N., Farris, P.W. and Freeland, J.R. (1994), "A model for determining retail product category assortment and shelf space allocation", *Decision Sciences*, Vol. 25 No. 3, pp. 359-84.

Bramel, J. and Simchi-Levi, D. (1997), *The Logic of Logistics: Theory, Algorithms, and Applications for Logistics Management*, Springer-Verlag, New York, NY.

Bultez, A. and Naert, P. (1988), "SHARP: shelf allocation for retailers' profit", *Marketing Science*, Vol. 7 No. 3, pp. 211-31.

Chung, K.-J., Chu, P. and Lan, S.-P. (2000), "A note on EOQ models for deteriorating items under stock dependent selling rate", *European Journal of Operational Research*, Vol. 124 No. 3, pp. 550-9.

Corstjens, M. and Doyle, P. (1981), "A model for optimizing retail space allocations", *Management Science*, Vol. 27 No. 7, pp. 822-33.

Desmet, P. and Renaudin, V. (1998), "Estimation of product category sales responsiveness to allocated shelf space", *International Journal of Research in Marketing*, Vol. 15 No. 5, pp. 443-57.

Drèze, X., Hoch, S.J. and Purk, M.E. (1994), "Shelf management and space elasticity", *Journal of Retailing*, Vol. 70 No. 4, pp. 301-26.

Dubelaar, C., Chow, G. and Larson, P.D. (2001), "Relationships between inventory, sales and service in a retail chain store operation", *International Journal of Physical Distribution & Logistics Management*, Vol. 31 No. 2, pp. 96-108.

Ellram, L.M., La Londe, B.J. and Weber, M.M. (1999), "Retail logistics", *International Journal of Physical Distribution & Logistics Management*, Vol. 29 No. 7/8, pp. 477-94.

Gerchak, Y. and Wang, Y. (1994), "Periodic-review inventory models with inventory-level-dependent demand", *Naval Research Logistics*, Vol. 41 No. 1, pp. 99-116.

Johnson, L.A. and Montgomery, D.C. (1974), *Operations Research in Production Planning, Scheduling, and Inventory Control*, Wiley, New York, NY.

Khouja, M. (1999), "The single-period (news-vendor) problem: literature review and suggestions for future research", *Omega*, Vol. 27 No. 5, pp. 537-53.

Larson, P.D. and DeMarais, R.A. (1999), "Psychic stock: an independent variable category of inventory", *International Journal of Physical Distribution & Logistics Management*, Vol. 29 No. 7/8, pp. 495-507.

Lau, H.-S. and Lau, A.H.-L. (1996), "The newsstand problem: a capacitated multiple-product single-period inventory problem", *European Journal of Operational Research*, Vol. 94 No. 1, pp. 29-42.

Lilien, G.L., Kotler, P. and Moorthy, K.S. (1992), *Marketing Models*, Prentice-Hall, Englewood Cliffs, NJ.

Mandal, B.N. and Phaujdar, S. (1989), "An inventory model for deteriorating items and stock-dependent consumption rate", *Journal of the Operational Research Society*, Vol. 40 No. 5, pp. 483-88.

Padmanabhan, G. and Vrat, P. (1995), "EOQ models for perishable items under stock-dependent selling rate", *European Journal of Operational Research*, Vol. 86 No. 2, pp. 281-92.

Ray, J., Goswami, A. and Chaudhuri, K.S. (1998), "On an inventory model with two levels of storage and stock-dependent demand rate", *International Journal of Systems Science*, Vol. 29 No. 3, pp. 249-54.

Schary, P.B. and Becker, B.W. (1972), "Distribution and final demand: the influence of availability", *Mississippi Valley Journal of Business and Economics*, Vol. 8 No. 1, pp. 17-26.

Smith, S.A. and Agrawal, N. (2000), "Management of multi-item inventory systems with demand substitution", *Operations Research*, Vol. 48 No. 1, pp. 50-64.

Urban, T.L. (1998), "An inventory-theoretic approach to product assortment and shelf-space allocation", *Journal of Retailing*, Vol. 74 No. 1, pp. 15-35.

Urban, T.L. and Baker, R.C. (1997), "Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns", *European Journal of Operational Research*, Vol. 103 No. 3, pp. 573-83.

Wang, Y. and Gerchak, Y. (2001), "Supply chain coordination when demand is shelf-space dependent", *Manufacturing & Service Operations Management*, Vol. 3 No. 1, pp. 82-7.

Whitin, T.M. (1957), *The Theory of Inventory Management*, Princeton University Press, Princeton, NJ.

Wolfe, H.B. (1968), "A model for control of style merchandise", *Industrial Management Review*, Vol. 9 No. 2, pp. 69-82.

Zurfryden, F.S. (1986), "A dynamic programming approach for product selection and supermarket shelf-space allocation", *Journal of the Operational Research Society*, Vol. 37 No. 4, pp. 413-22.

Appendix. The derivation of the solution to the displayed news-vendor problem

The profit function is a function of q and s and is expressed as (from (equation 6)):

$$\pi = (p-c)q - vs - (p+h)\left[s^{\beta} - \alpha\beta(T - \frac{q-s}{\alpha s^{1-\beta}})\right]^{1/\beta}.$$

Taking the derivative of π with respect to q:

$$\frac{\partial \pi}{\partial q} = (p-c) - (p+h)s^{\beta-1}\left[s^{\beta} - \alpha\beta(T - \frac{q-s}{\alpha s^{1-\beta}})\right]^{\frac{1-\beta}{\beta}}. \tag{A-1}$$

Setting $\frac{\partial \pi}{\partial q} = 0$ provides:

$$\left(\frac{p-c}{p+h}\right)s^{1-\beta} = \left[s^{\beta} - \alpha\beta(T - \frac{q-s}{\alpha s^{1-\beta}})\right]^{\frac{1-\beta}{\beta}}. \tag{A-2}$$

Let $\theta = \frac{p-c}{p+h}$ and, solving for q:

$$q = \alpha s^{1-\beta}T - s\left[\frac{(1-\beta) - \theta^{\beta}}{\beta}\right]. \tag{A-3}$$

Taking the derivative of π with respect to s:

$$\frac{\partial \pi}{\partial s} = -v + \frac{(p+h)(1-\beta)(q-s)}{s^{2-\beta}}\left[s^{\beta} - \alpha\beta(T - \frac{q-s}{\alpha s^{1-\beta}})\right]^{\frac{1-\beta}{\beta}}. \tag{A-4}$$

Setting $\frac{\partial \pi}{\partial s} = 0$ provides:

$$\frac{vs^{2-\beta}}{(p+h)(1-\beta)(q-s)} = \left[s^{\beta} - \alpha\beta(T - \frac{q-s}{\alpha s^{1-\beta}}\right]^{\frac{1-\beta}{\beta}}. \tag{A-5}$$

From (A-2), this is equivalent to:

$$\frac{vs^{2-\beta}}{(p+h)(1-\beta)(q-s)} = \left(\frac{p-c}{p+h}\right)s^{1-\beta}. \tag{A-6}$$

Simplifying:

$$s = \frac{(p-c)(1-\beta)q}{(p-c)(1-\beta)+v}. \tag{A-7}$$

From (A-3), this is equivalent to:

$$s = \frac{(p-c)(1-\beta)}{(p-c)(1-\beta)+v}\left\{\alpha s^{1-\beta}T - s\left[\frac{(1-\beta)-\theta^{\beta}}{\beta}\right]\right\}. \tag{A-8}$$

Let $\phi = (1-\beta)(p-c)$ and, solving for s:

$$s = \left[\frac{\alpha\beta\phi T}{\phi(1-\theta^{\beta}) + \beta v}\right]^{1/\beta}. \tag{A-9}$$